Chapter 9
Operators

Mathcad includes ordinary arithmetic operators like + and /, matrix operators like transpose and determinant, and special operators like iterated sum, iterated product, derivatives, and integrals. This chapter provides an overview of Mathcad's operators that can be evaluated numerically.

This chapter contains the following sections:

Working with operators

Inserting operators and operands, and evaluating expressions with operators.

Arithmetic and Boolean operators

How to perform basic arithmetic in Mathcad. How to use Boolean operators such as “>” and “<.”

Vector and matrix operators

Vector and matrix arithmetic, as well as specialized array operations.

Summations and products

How to use Mathcad's summation and product operators.

Derivatives

How to use Mathcad's derivative operators.

Integrals

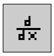
Introduction to Mathcad's definite integral operator, which automatically applies the most accurate of several numerical integration algorithms via the AutoSelect feature.

Pro **Customizing operators**

How to define your own operators.

Working with operators

Inserting an operator

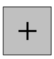
You insert the common arithmetic operators into math expressions in Mathcad using the standard keystrokes, like ***** and **+**, that you use in spreadsheet and other applications. But all of Mathcad's operators can be inserted into math expressions by clicking buttons in the math toolbars. For example, you insert Mathcad's derivative operator by clicking  on the Calculus toolbar, or by typing **?**. Choose **Toolbars** from the **View** menu to see any of the math toolbars. See "Operators" on page 334 in the Appendices for a complete list of operators.

Note In general, the math toolbars work only in blank space in your worksheet or when you have already clicked in a math region. To use the math toolbars in text, first click in the text and choose **Math Region** from the **Insert** menu. This creates a math placeholder in the text into which you can insert operators using the math toolbars.

Tip You can find out the keyboard shortcut for inserting an operator by hovering the mouse pointer over an operator button in one of the Math toolbars and reading the tooltip that appears.

As introduced in Chapter 4, "Working with Math," when you insert a Mathcad operator into a blank space in your worksheet, a mathematical symbol with empty *placeholders* appears in the worksheet. The placeholders are for you to enter expressions that are the *operands* of the operator. The number of empty placeholders varies with the operator: some operators like the factorial operator have only a single placeholder, while others such as the definite integral have several. You must enter a valid math expression in all of the placeholders of an operator to calculate a result with that operator.

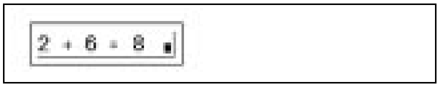
Here is a very simple example involving Mathcad's addition operator:

- Click in a blank space in your worksheet and click  on the Arithmetic toolbar, or simply type **+**. The addition operator with two placeholders appears.

- Enter **2** in the first placeholder.

- Click in the second placeholder, or press [**Tab**] to move the cursor, and enter **6**.

- Press =, or click on the Evaluation toolbar, to see a numerical result.



Tip See Chapter 4, “Working with Math,” for a discussion of how to build and edit more complex math expressions, including how to use the *editing lines* to specify what becomes the operand of the next operator you insert or delete.

Additional operators

This chapter focuses on those Mathcad operators you can use to calculate numerical answers. Additional operators in Mathcad include:

- *Symbolic operators*, which can only be used to generate other math expressions or exact numerical answers. As described in Chapter 14, “Symbolic Calculation,” Mathcad’s symbolic processor understands virtually any Mathcad expression, but expressions that include the following operators on the Calculus toolbar can *only* be evaluated symbolically: indefinite integral, two-sided limit, limit from above, and limit from below. To evaluate an expression symbolically, click on the Evaluation toolbar.

Pro

- *Programming operators*, which you use to link multiple Mathcad expressions via conditional branching, looping constructs, local scoping of variables, and other attributes of traditional programming languages. These operators, available only in Mathcad Professional (click on the Math toolbar), are introduced in Chapter 15, “Programming.”

Arithmetic and Boolean operators

Arithmetic operators

You can freely combine all types of numbers with arithmetic operators you access on the Arithmetic toolbar. Figure 9-1 shows examples.

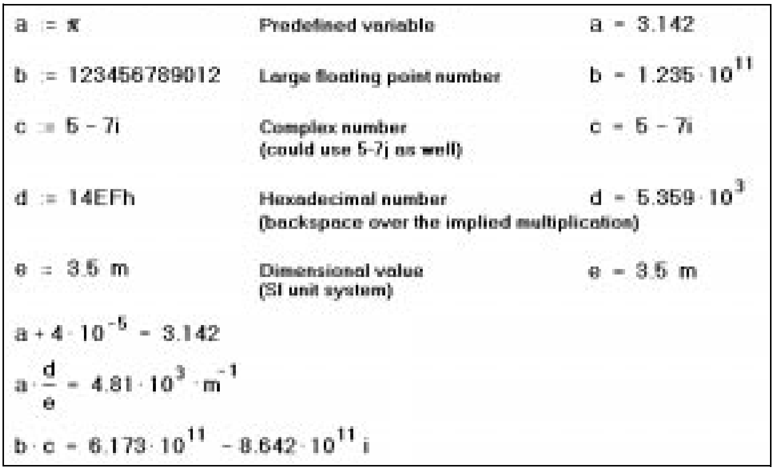


Figure 9-1: Combining different types of numbers with arithmetic operators.

Boolean operators

Mathcad includes logical or *Boolean* operators on the Evaluation toolbar. Unlike other operators, the Boolean operators can return only a zero or a one. Despite this, they can be very useful to perform tests on your expressions.

The following table lists the Boolean operators available on the Evaluation toolbar and their meaning with numbers. Note that the Boolean equal sign (bold equal sign) is different from the evaluation equal sign you insert by typing **=**.

Appearance	Button	Description
$w = z$	**=**	Boolean equal; displays as bold equal sign.
$x > y$	>	Greater than.
$x < y$	<	Less than.
$x \geq y$	≥	Greater than or equal to.
$x \leq y$	≤	Less than or equal to.
$w \neq z$	≠	Not equal to.

Note The Boolean operators return 1 if the expression is true, 0 otherwise. The four operators >, <, ≤ , and ≥ cannot take complex numbers because the concepts of greater than and less than lose their meaning in the complex plane.

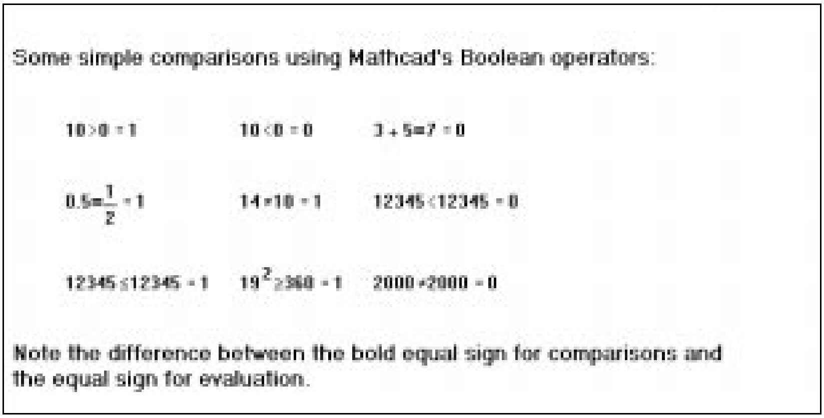


Figure 9-2: Using boolean operators.

Tip Boolean operators can also be used to compare *strings*. Mathcad compares two strings character by character by determining the ASCII codes of the characters. For example, the string “Euler” precedes the string “Mach” in ASCII order and so the expression (“Euler”<“Mach”) evaluates to 1. To determine the character ordering Mathcad uses in comparing strings, see “ASCII codes” on page 353 in the Appendices.

Complex operators

Mathcad has the following arithmetic operators for working with complex numbers:

Appearance	**Button**	**Description**
$\bar{z}$		Complex conjugate of z. To apply the conjugate operator to an expression, select the expression, then press the double-quote key (**"**). The conjugate of the complex number $a + b \cdot i$ is $a - b \cdot i$.
$\|z\|$	|×|	The magnitude of the number z.

Figure 9-3 shows some examples of how to use complex numbers in Mathcad.

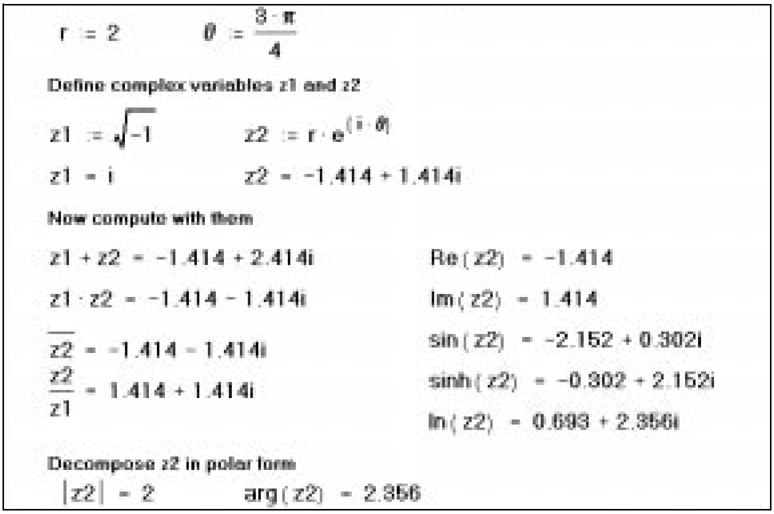


Figure 9-3: Complex numbers in Mathcad.

Vector and matrix operators

Most of the operators on the Arithmetic toolbar also have meaning for vectors and matrices. For example, when you use the addition operator to add two arrays of the same size, Mathcad performs the standard element-by-element addition. Mathcad also uses the conventional arithmetic operators for matrix subtraction, matrix multiplication, integer powers, and determinants, among others.

Some of Mathcad's operators have special meanings for vectors and matrices, and many of these are grouped on the Matrix toolbar (click on the Math toolbar). For example, the multiplication symbol means multiplication when applied to two numbers, but it means dot product when applied to vectors, and matrix multiplication when applied to matrices.

The table below describes Mathcad's vector and matrix operations. Operators not listed in this table do not work for vectors and matrices. You can, however, use the vectorize operator (click on the Matrix toolbar) to perform any scalar operation or function element by element on a vector or matrix. See "Doing calculations in parallel" on page 229. Figure 9-4 shows some ways to use vector and matrix operators.

In the following table,

- **A** and **B** represent arrays, either vector or matrix.
- **u** and **v** represent vectors.

- **M** represents a square matrix.
- u_i and v_i represent the individual elements of vectors **u** and **v**.
- z represents a scalar.
- m and n represent integers.

Appearance	Button	Description
$\mathbf{A} \cdot z$	$\vec{x} \cdot \vec{y}$	Scalar multiplication. Multiplies each element of **A** by the scalar z.
$\mathbf{u} \cdot \mathbf{v}$	$\vec{x} \cdot \vec{y}$	Dot product. Returns a scalar: $\Sigma\langle u_i \cdot v_i \rangle$. The vectors must have the same number of elements.
$\mathbf{A} \cdot \mathbf{B}$	$\vec{x} \cdot \vec{y}$	Matrix multiplication. Returns the matrix product of **A** and **B**. The number of columns in **A** must match the number of rows in **B**.
$\mathbf{A} \cdot \mathbf{v}$	$\vec{x} \cdot \vec{y}$	Vector/matrix multiplication. Returns the product of **A** and **v**. The number of columns in **A** must match the number of rows in **v**.
$\dfrac{\mathbf{A}}{z}$	÷	Scalar division. Divides each element of the array **A** by the scalar z.
$\mathbf{A} + \mathbf{B}$	+	Vector and matrix addition. Adds corresponding elements of **A** and **B**. The arrays **A** and **B** must have the same number of rows and columns.
$\mathbf{A} + z$	+	Scalar addition. Adds z to each element of **A**.
$\mathbf{A} - \mathbf{B}$	–	Vector and matrix subtraction. Subtracts corresponding elements of **A** and **B**. The arrays **A** and **B** must have the same number of rows and columns.
$\mathbf{A} - z$	–	Scalar subtraction. Subtracts z from each element of **A**.
$-\mathbf{A}$	–	Negative of vector or matrix. Returns an array whose elements are the negatives of the elements of **A**.
$\mathbf{M}^n$	x^Y	nth power of square matrix **M** (using matrix multiplication). n must be an integer. $\mathbf{M}^{-1}$ represents the inverse of **M**. Other negative powers are powers of the inverse. Returns a matrix.
$\lvert\mathbf{v}\rvert$	$\lvert x \rvert$	Magnitude of vector. Returns $\sqrt{\mathbf{v} \cdot \bar{\mathbf{v}}}$ where $\bar{\mathbf{v}}$ is the complex conjugate of **v**.
$\lvert\mathbf{M}\rvert$	$\lvert x \rvert$	Determinant. **M** must be a square matrix.
$\mathbf{A}^{\mathrm{T}}$	M^T	Transpose. Interchanges row and columns of **A**.

$\mathbf{u} \times \mathbf{v}$	Cross product. **u** and **v** must be three-element vectors; result is another three-element vector.
$\overline{\mathbf{A}}$	Complex conjugate. Takes complex conjugate of each element of **A**. Insert in math with the double-quote key (**"**).
$\Sigma\mathbf{v}$	Vector sum. Sum elements in **v**.
$\overrightarrow{\mathbf{A}}$	Vectorize. Treat all operations in **A** element by element. See "Doing calculations in parallel" on page 229 for details.
$\mathbf{A}^{\langle n \rangle}$	Array superscript. *n*th column of array **A**. Returns a vector.
v_n	Vector subscript. *n*th element of a vector.
$A_{m,\,n}$	Matrix subscript. *m*,*n*th element of a matrix.

Tip Operators and functions that expect vectors always expect column vectors. They do not apply to row vectors. To change a row vector into a column vector, use the transpose operator by clicking M^T on the Matrix toolbar.

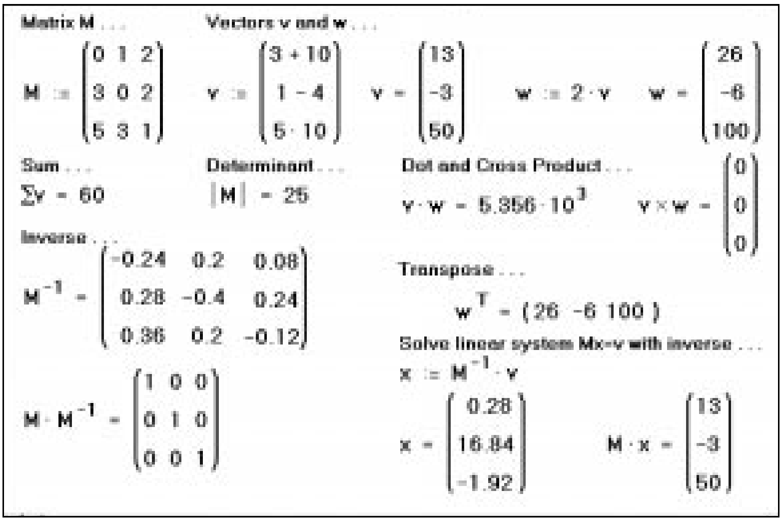


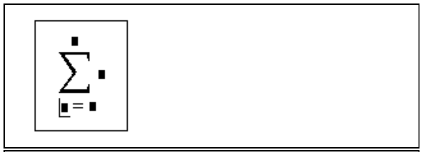
Figure 9-4: Vector and matrix operations.

Summations and products

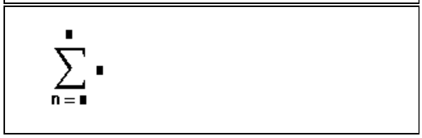
The summation operator sums an expression over all values of an index. The iterated product operator works much the same way. It takes the product of an expression over all values of an index.

To create a summation operator in your worksheet:

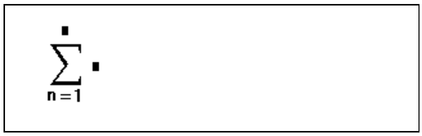
■ Click in a blank space. Then click  on the Calculus toolbar. A summation sign with four placeholders appears.



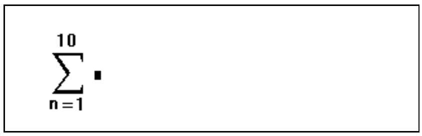
■ Type a variable name in the placeholder to the left of the equal sign. This variable is the index of summation. It is defined only within the summation operator and therefore has no effect on, and is not influenced by, variable definitions outside the summation operator.



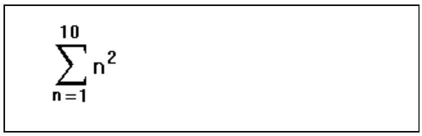
■ Type an integer, or any expression that evaluates to an integer, in the placeholder to the right of the equal sign.

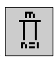


■ Type an integer, or any expression that evaluates to an integer, in the single placeholder above the sigma.



■ Type the expression you want to sum in the remaining placeholder. Usually, this expression involves the index of summation. If this expression has several terms, first type an apostrophe (`'`) to create parentheses around the placeholder.



Iterated products are similar to summations. Just click  on the Calculus toolbar and fill in the placeholders as described earlier.

Tip Use the keyboard shortcut [**Ctrl**][**Shift**]**4** to enter the iterated sum and the shortcut [**Ctrl**][**Shift**]**3** to enter the iterated product operator.

Figure 9-5 shows some examples of how to use the summation and product operators.

To evaluate multiple summations, place another summation in the final placeholder of the first summation. An example appears at the bottom of Figure 9-5.

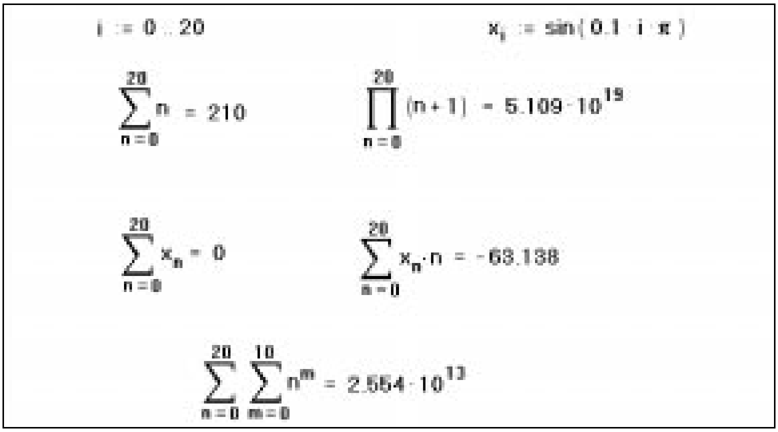
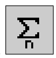
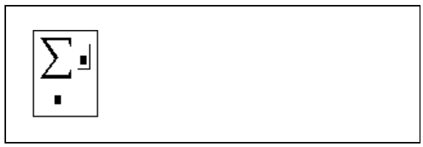


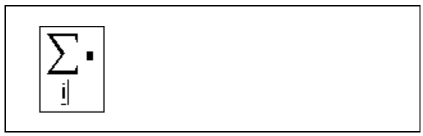
Figure 9-5: Summations and products.

When you use the summation operator shown in Figure 9-5, the summation must be carried out over integers and in steps of one. Mathcad provides more general versions of these operators that can use any range variable you define as an index of summation. To use these operators, first define a range variable, for example, by typing **`i:1,2;10`**. Then do the following:

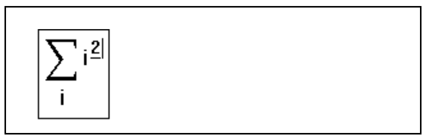
- Click in a blank space. Then click  on the Calculus toolbar. A summation sign with two placeholders appears.

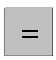


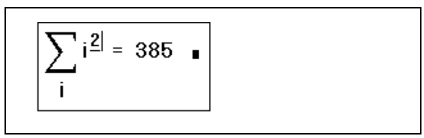
- Click on the bottom placeholder and type the name of a range variable. The range variable you use here must have been defined previously in the worksheet.



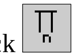
- Click on the placeholder to the right of the summation sign and type an expression involving the range variable. If this expression has several terms, first type an apostrophe (`'`) to create parentheses around the placeholder.

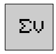


- Press =, or click  on the Evaluation toolbar, to get a result.



Tip If you don't want to take the time to click in each placeholder, you can quickly enter the previous expression, for example, by typing **i$i^2**.

A generalized version of the iterated product also exists. To use it, click  on the Calculus toolbar. Then fill in the two placeholders.

Tip The operation of summing the elements of a vector is so common that Mathcad provides a special operator for it. The vector sum operator (click  on the Matrix toolbar) sums the elements of a vector without needing a range variable.

Variable upper limit of summation

Mathcad's range summation operator runs through each value of the range variable you place in the bottom placeholder. It is possible, by judicious use of Boolean expressions, to sum only up to a particular value. In Figure 9-6, the term $i \le x$ returns the value 1 whenever it is true and 0 whenever it is false. Although the summation operator still sums over each value of the index of summation, those terms for which $i > x$ are multiplied by 0 and hence do not contribute to the summation.

You can also use the four-placeholder summation and product operators to compute sums and products with a variable upper limit, but note that the upper limit in these operators must be an integer.

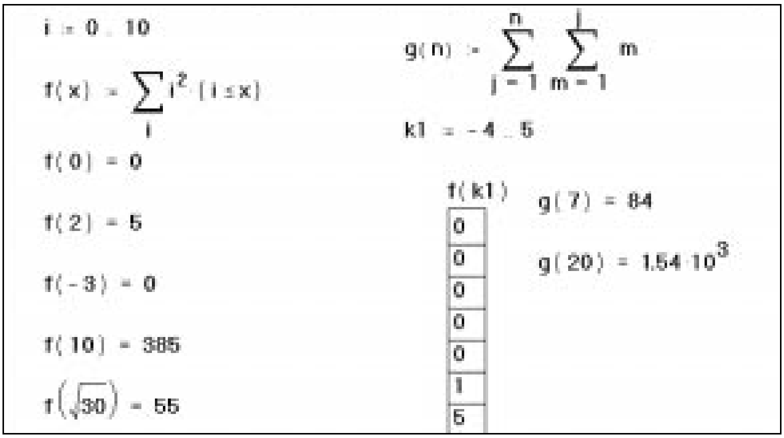


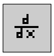
Figure 9-6: A variable upper limit of summation.

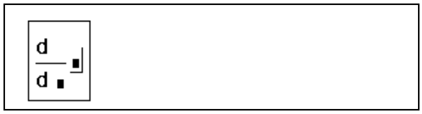
Derivatives

You can use Mathcad's derivative operators to evaluate the first or high order derivatives of a function at a particular point.

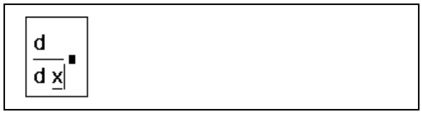
As an example, here's how to evaluate the first derivative of x^3 with respect to x at the point $x = 2$:

- First define the point at which you want to evaluate the derivative. As a shortcut, type **x:2**.

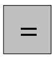
- Click below the definition of *x*. Then click  on the Calculus toolbar. A derivative operator appears with two placeholders.



- Click on the bottom placeholder and type **x**. You are differentiating with respect to this variable.



- Click on the placeholder to the right of the d/dx and enter **x^3**. This is the expression to be differentiated.

- Press =, or click  on the Evaluation toolbar, to get the result.

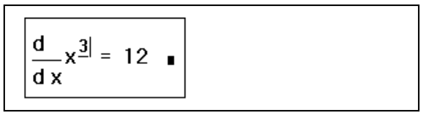


Figure 9-7 shows examples of differentiation in Mathcad.

With Mathcad's derivative algorithm, you can expect the first derivative to be accurate within 7 or 8 significant digits, provided that the value at which you evaluate the derivative is not too close to a singularity of the function. The accuracy of this algorithm tends to decrease by one significant digit for each increase in the order of the derivative (see "Derivatives of higher order" on page 162).

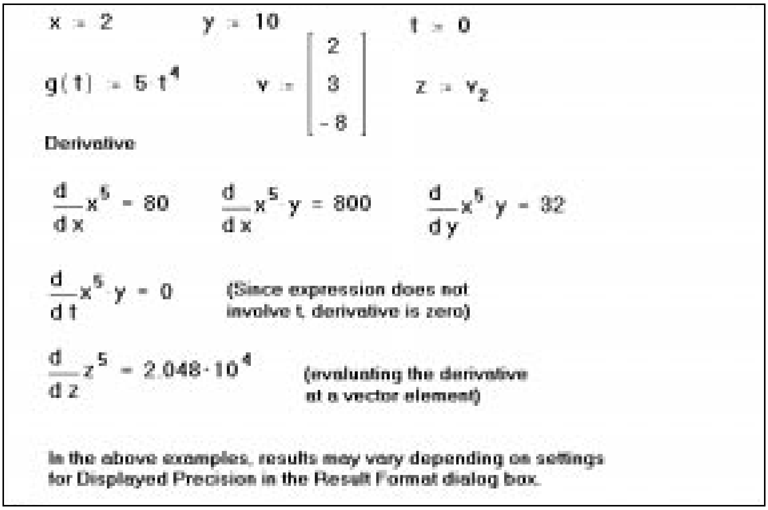


Figure 9-7: Examples of Mathcad differentiation.

Note Keep in mind that the result of numerical differentiation is not a function, but a single number: the computed derivative at the indicated value of the differentiation variable. In the previous example, the derivative of x^3 is not the expression $3x^2$ but $3x^2$ evaluated at $x = 2$. To evaluate derivatives symbolically, see Chapter 14, “Symbolic Calculation.”

Although differentiation returns just one number, you can still define one function as the derivative of another. For example:

$$f(x) := \frac{d}{dx}g(x)$$

Evaluating $f(x)$ returns the numerically computed derivative of $g(x)$ at x.

You can use this technique to evaluate the derivative of a function at many points. An example of this is shown in Figure 9-8.

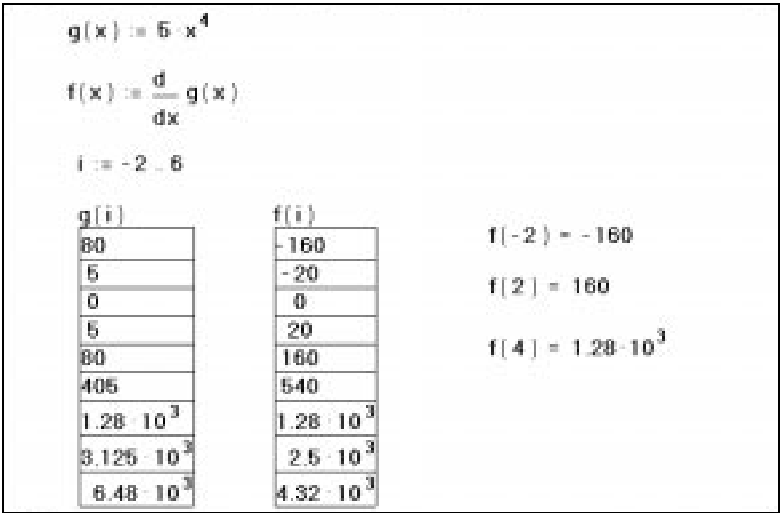


Figure 9-8: Evaluating the derivative of a function at several points.

There are some important things to remember about differentiation in Mathcad:

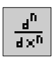
- The expression to be differentiated can be either real or complex.
- The differentiation variable must be a single variable name. If you want to evaluate the derivative at several different values stored in a vector, you must evaluate the derivative at each individual vector element (see Figure 9-7).

Derivatives of higher order

Mathcad has an additional derivative operator for evaluating the *n*th order derivative of a function at a particular point.

As an example, here's how to evaluate the third derivative of x^9 with respect to x at the point $x = 2$:

- First define the point at which you want to evaluate the derivative. As a shortcut, type **x:2**.

- Click below the definition of *x*. Then click  on the Calculus toolbar. A derivative operator appears with four placeholders.

- Click on the bottom-most placeholder and type **x**. You are differentiating with respect to this variable.

- Click on the expression above and to the right of the previous placeholder and type **`3`**. This must be an integer between 0 and 5 inclusive. Note that the placeholder in the numerator automatically mirrors whatever you've typed.

$$\frac{d^3}{dx^3}\blacksquare$$

- Click on the placeholder to the right of the d/dx and type **`x^9`**. This is the expression to be differentiated.

$$\frac{d^3}{dx^3}x^9$$

- Press =, or click = on the Evaluation toolbar, to see the result.

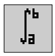
$$\frac{d^3}{dx^3}x^9 = 3.226 \cdot 10^4 \ \blacksquare$$

Note For $n = 1$, the nth derivative operator gives the same answer as the first-derivative operator discussed on page 160.

Integrals

You can use Mathcad's integral operator to numerically evaluate the definite integral of a function over some interval.

As an example, here's how to evaluate the definite integral of $\sin^2(x)$ from 0 to $\pi/4$. (In Mathcad you enter $\sin^2(x)$ as $\sin(x)^2$.) Follow these steps:

- Click in a blank space and click  on the Calculus toolbar. An integral symbol appears, with placeholders for the integrand, limits of integration, and variable of integration.

$$\int_{\blacksquare}^{\blacksquare} \blacksquare \, d\blacksquare$$

- Click on the bottom placeholder and type **`0`**. Click on the top placeholder and type **`p[Ctrl]G/4`**. These are the upper and lower limits of integration.

$$\int_{0}^{\frac{\pi}{4}} \blacksquare \, d\blacksquare$$

- Click on the placeholder between the integral sign and the "d." Then type **`sin(x)^2`**. This is the expression to be integrated.

$$\int_0^{\frac{\pi}{4}} \sin(x)^2 \, d\blacksquare$$

- Click on the remaining placeholder and type **`x`**. This is the variable of integration.

  Then press =, or click = on the Evaluation toolbar, to see the result

$$\int_0^{\frac{\pi}{4}} \sin(x)^2 \, dx = 0.143$$

Note Some points to keep in mind when you evaluate integrals in Mathcad: The limits of integration must be real. The expression to be integrated can, however, be either real or complex. Except for the integrating variable, all variables in the integrand must have been defined elsewhere in the worksheet. The integrating variable must be a single variable name. If the integrating variable involves units, the upper and lower limits of integration must have the same units.

Integration algorithms and AutoSelect

Mathcad has a number of numerical integration methods at its disposal to calculate the numerical approximation of an integral. When you evaluate an integral, by default Mathcad uses an *AutoSelect* procedure to choose the most accurate integration method. If you have Mathcad Professional, you can override the integration AutoSelect and choose from among the available algorithms yourself.

Here are the methods from which Mathcad chooses from when you evaluate an integral numerically:

Romberg
Applies a default Romberg integration method that divides the interval of integration into equally spaced subintervals.

Adaptive
Applies an adaptive quadrature algorithm in cases where the integrand varies considerably in magnitude over the interval of integration.

Infinite Limit
Applies an algorithm designed for improper integral evaluation in cases where either limit of integration is ∞ or $-\infty$.

Singular Endpoints
Applies a routine that avoids use of the interval endpoints in cases where the integrand is undefined at either limit of integration.

Note Although designed to handle a wide range of problems, Mathcad's integration algorithms—like all numerical methods—can have difficulty with ill-behaved integrands. If the expression to be integrated has singularities or discontinuities, for example, the solution may still be inaccurate.

Pro With Mathcad Professional, you can override Mathcad's integration AutoSelect as follows:

- Evaluate the value of the integral as described on page 163, allowing Mathcad to AutoSelect an integration algorithm.
- Click with the right mouse button on the integral, and remove the check from **AutoSelect** on the pop-up menu.
- Check one of the listed integration methods on the pop-up menu. Mathcad recalculates the integral using the method you selected.

Tip In some cases, you may be able to find an exact numerical value for your integral by using Mathcad's symbolic integration capability. You can also use this capability to evaluate *indefinite* integrals. See Chapter 14, "Symbolic Calculation."

Variable limits of integration

Although the result of an integration is a single number, you can always use an integral with a range variable to obtain results for many numbers at once. You might do this, for example, when you set up a variable limit of integration. Figure 9-9 shows how to do this.

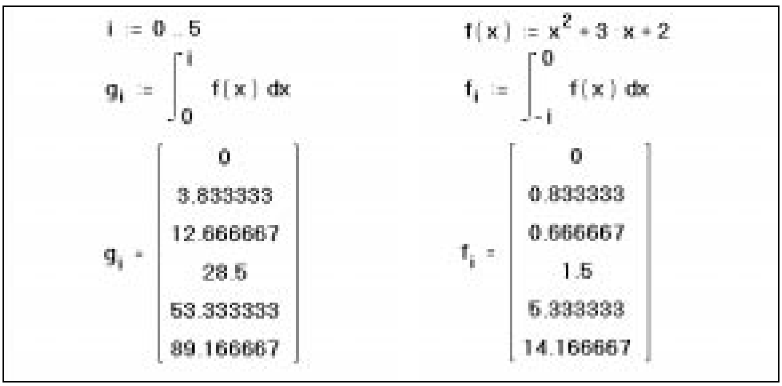


Figure 9-9: Variable limits of integration.

Keep in mind that calculations such as those shown in Figure 9-9 require repeatedly evaluating an integral. This may take considerable time depending on the complexity of the integrals, the length of the interval, and the value of the tolerance parameter TOL (see below).

Tolerance for integrals

Mathcad's numerical integration algorithms make successive estimates of the value of the integral and return a value when the two most recent estimates differ by less than the value of the built-in variable TOL.

As described in "Built-in variables" on page 121, you can change the value of the tolerance by including definitions for TOL directly in your worksheet. You can also change the tolerance by using the Built-In Variables tab when you choose **Options** from the **Math** menu. To see the effect of changing the tolerance, choose **Calculate Document** from the **Math** menu to recalculate all the equations in the worksheet.

If Mathcad's approximation to an integral fails to converge to an answer, Mathcad marks the integral with an error message. Failure to converge can occur when the function has singularities or "spikes" in the interval or when the interval is extremely long.

Note When you change the tolerance, keep in mind the trade-off between accuracy and computation time. If you decrease (tighten) the tolerance, Mathcad computes integrals more accurately, but Mathcad takes longer to return a result. Conversely, if you increase (loosen) the tolerance, Mathcad computes more quickly, but the answers are less accurate.

Contour integrals

You can use Mathcad to evaluate complex contour integrals. To do so, first parametrize the contour and then integrate over the parameter. If the parameter is something other than arc length, you must also include the derivative of the parametrization as a correction factor. Figure 9-10 shows an example. Note that the imaginary unit *i* used in specifying the path must be typed as **`1i`**.

$$x(t) := 2 \cdot \cos(t) \qquad y(t) := 2 \cdot \sin(t)$$

Path: $z(t) := x(t) + i \cdot y(t)$

Function to integrate: $f(z) := \frac{1}{z}$

$$\int_0^{\pi} f(z(t)) \cdot \frac{d}{dt} z(t)\, dt = 3.142i$$

Figure 9-10: A complex contour integral in Mathcad.

Multiple integrals

You can also use Mathcad to evaluate double or multiple integrals. To set up a double integral, for example, click on the Calculus toolbar twice. Fill in the integrand, the limits, and the integrating variable for each integral. Figure 9-11 shows an example.

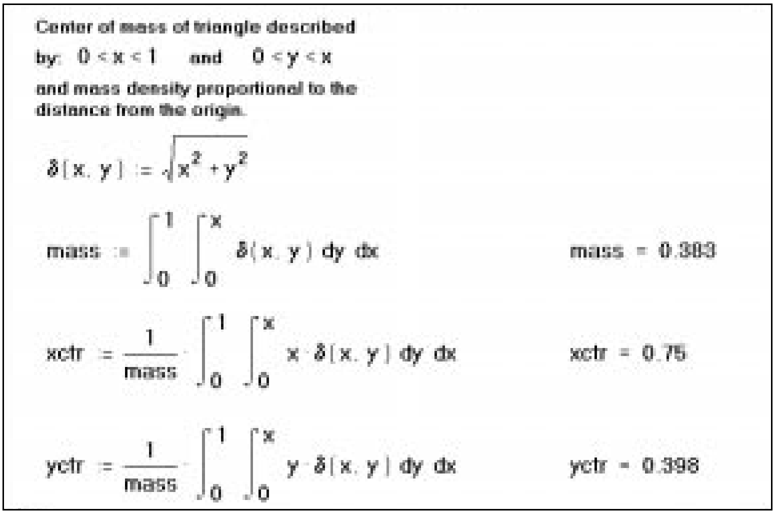


Figure 9-11: Double integrals.

Note Multiple integrals generally take much longer to converge to an answer than single integrals. Wherever possible, use an equivalent single integral in place of a multiple integral.

Customizing operators

Pro This section describes how you can use Mathcad Professional to define and use your own customized operators.

You can think of operators and functions as being fundamentally very similar. A function takes “arguments” and returns a result. An operator, likewise, takes “operands” and returns a result. The differences are largely cosmetic:

- Functions usually have names you can spell, like *tan* or *spline*; operators are generally math symbols like “+” or “×”.
- Arguments to a function are enclosed by parentheses, they come after the function’s name, and they’re separated by commas. Operands, on the other hand, can appear

elsewhere. For example, you'll often see $f(x, y)$ but you'll rarely see $x\,f\,y$. Similarly, you'll often find "$x + y$" but you rarely find "$+(x, y)$".

Defining a custom operator

You define a custom operator just as if you were defining a function that happens to have an unusual looking name:

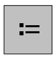
- Type the operator name followed by a pair of parentheses. Enter the operands (two at the most) between the parentheses.
- Enter the definition symbol **:=**.
- Type an expression describing what you want the operator to do with its operands on the other side of the definition symbol.

Tip Mathcad provides for you a collection of math symbols to define custom operators. To access these symbols, open the QuickSheets from the Resource Center (choose **Resource Center** on the **Help** menu) and then click on "Math Symbols." You can drag any of these symbols to your worksheet for use in creatomg a new operator name.

For example, suppose you want to define a new division operator using the symbol "$\div$".

- Drag the symbol into your worksheet from the "Math Symbols" QuickSheet.

- Type a left parenthesis followed by two names separated by a comma. Complete this argument list by typing a right parenthesis.

- Press the colon (**:**) key, or click  on the Arithmetic toolbar. You see the definition symbol followed by a placeholder.

- Type the function definition in the placeholder.

At this point, you've defined a function which behaves in every way like the user-defined functions described in Chapter 8, "Calculating in Mathcad." You could, if you wanted to, type "$\div(1, 2) =$" in your worksheet and see the result "0.5" on the other side of the equal sign.

Tip Once you've defined the new operator, click on "Personal QuickSheets" in the QuickSheets of the Mathcad Resource Center. Then drag the definition into the QuickSheet. When you need to use this operator again, just open your Personal QuickSheet and drag it into a new worksheet.

Using a custom operator

Once you've defined a new operator, you can use it in your calculations just as you would use any of Mathcad's built-in operators. The procedure for using a custom operator depends on whether the operator has one operand (like "-1" or "5!") or two (like "$1 \div 2$").

To insert an operator having two operands:

- Click xfy on the Evaluation toolbar. You'll see three empty placeholders.

- In the middle placeholder, insert the name of the operator. Alternatively, copy the name from the operator definition and paste it into the placeholder.

- In the remaining two placeholders, enter the two operands.

- Press =, or click = on the Evaluation toolbar, to get the result.

Tip An alternative way to display an operator having two operands is to click $x^f y$ on the Evaluation toolbar. If you follow the preceding steps using this operator, you'll see a tree-shaped display.

To insert an operator having only one operand, decide first whether you want the operator to appear before the operand, as in "-1," or after the operand as in "5!." The former is called a *prefix* operator; the latter is a *postfix* operator. The example below shows how to use a prefix operator. The steps for creating a postfix operator are almost identical.

Before you can reproduce the steps in the following example, you'll first have to define an operator "$\neg(x)$". To do so, follow the steps for defining $\div(x, y)$ in the previous section, substituting the symbol "$\neg$" for "$\div$" and using only one argument instead of two. Then, to evaluate with the new operator:

- Click f x on the Evaluation toolbar to make a *prefix* operator. Otherwise, click x f. In either case, you see two empty placeholders.

- If you clicked f x, put the operator name in the first placeholder. Otherwise put it in the second placeholder. In either case, you may find it more convenient to copy the name from the operator definition and paste it into the placeholder.

- In the remaining placeholder, place the operand.

- Press =, or click = on the Evaluation toolbar, to see the result.

Tip Just as Mathcad can display a custom operator as if it were a function, you can conversely display a function as if it were an operator. For example, many publishers prefer to omit parentheses around the arguments to certain functions (*sin* x rather than $\sin(x)$). To create this notation, you can treat the *sin* function as an operator with one operand.
